THE ORIENTAL INSTITUTE

OF THE UNIVERSITY OF CHICAGO

Engineering and Mathematics In Ancient Egypt

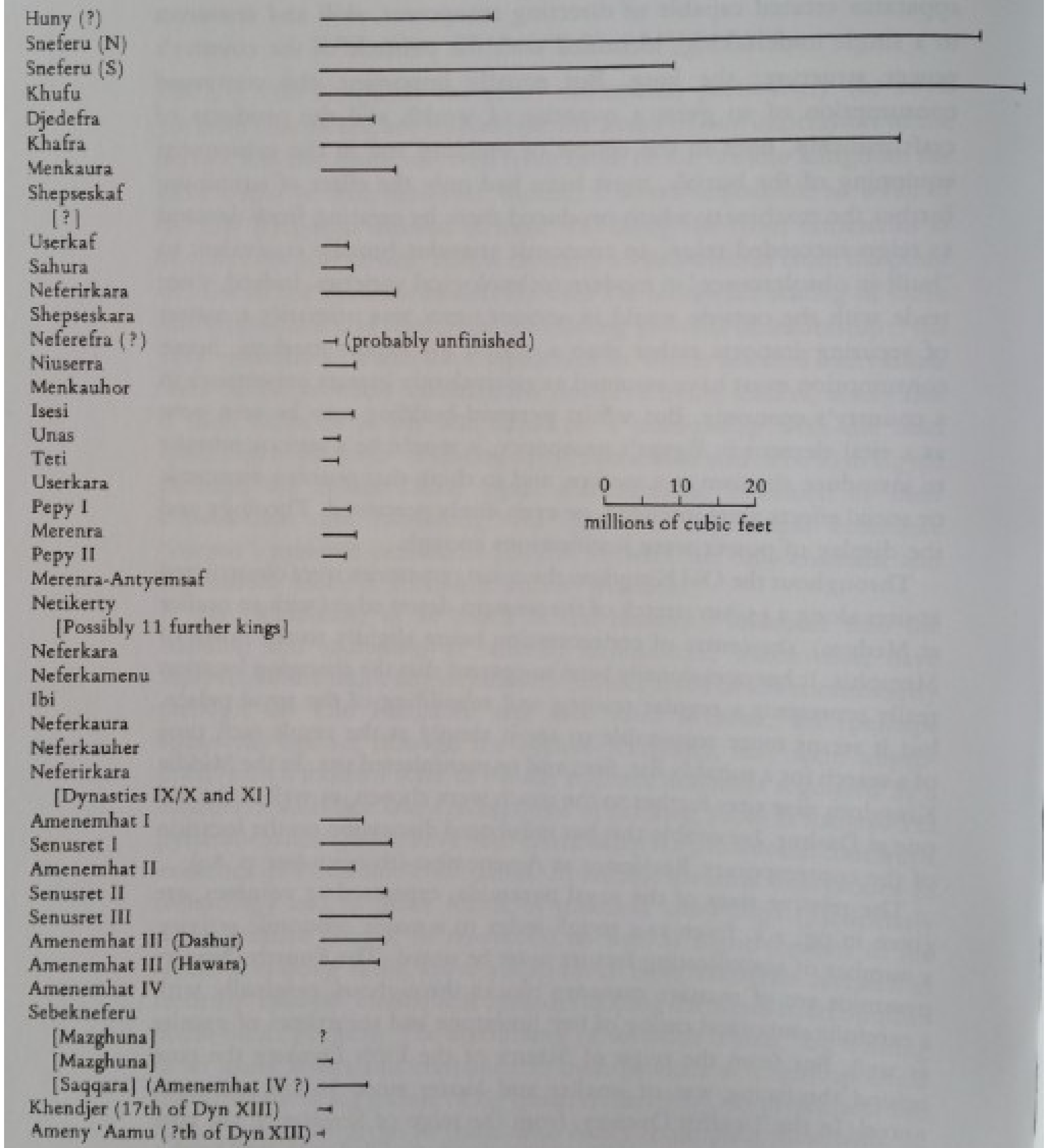
Joey Cross

Session 3
Teacher Cohort Academy 2016-2017

Engineering:
Stone Architecture

Egyptian Architecture and Engineering

- Stone Architecture
  - The oldest and largest in the world, beginning ca 2600 BCE
  - Coincides with the beginning of the Egyptian state


Huny (?)
Sneferu (N)
Sneferu (S)
Khufu
Djedefra
Khafra
Menkaura
Shepseskaf
[?]
Userkaf
Sahura
Neferirkara
Shepseskara
Neferefra (?)
(probably unfinished)
Niuserra
Menkauhor
Isesi
Unas
Teti
Userkara
Pepy I
Merenra
Pepy II
Merenra-Antyemsaf
Netikerty
[Possibly 11 further kings]
Neferkara
Neferkamenu
Ibi
Neferkaura
Neferkauher
Neferirkara
[Dynasties IX/X and XI]
Amenemhat I
Senusret I
Amenemhat II
Senusret II
Senusret III
Amenemhat III (Dashur)
Amenemhat III (Hawara)
Amenemhat IV
Sebekneferu
[Mazghuna]
?
[Mazghuna]
[Saqqara] (Amenemhat IV ?)
Khendjer (17th of Dyn XIII)
Ameny 'Aamu (?th of Dyn XIII)
0
10
20
millions of cubic feet

Egyptian Architecture and Engineering

- Stone Architecture
  - Most important types of stone:
    - Limestone
    - Sandstone
    - Granite

Egyptian Architecture and Engineering

- Stone Architecture
  - Was readily available in great quantities within Egypt itself
  - Limestone: from Cairo to just south of Luxor
  - Sandstone: from Luxor on south
  - Granite: widely distributed

Mediterranean Sea
Rosetta
Alexandria
Buto
Sebennytos
Mendes
Tanis
Avaris
Naucratis
Busiris
Leontopolis
Athribis
Bubastis
Lower Egypt
Sinai
Heliopolis
Giza
CAIRO
Abusir
Memphis
Saqqara
Dahshur
FAIYUM
Lisht
Meydum
Hawara
Lahun
Herakleopolis
el-Hiba
Beni Hassan
Hermopolis
el-Amarna
Red Sea
The Nile River
Dendera
Abydos
Naqada
Coptos
Thebes
Tod
Gebelein
Esna
el-Kab
Hierakonpolis
Edfu
Upper Egypt
Gebel el-Silsil
Sehel Island
Elephantine
1st Cataract
Philae
Beit el Wali
Amada
Aniba
Derr
Abu Simbel

Egyptian Architecture and Engineering

- Stone Architecture
  - Quality and symbolism were important
    - Monumental architecture itself was symbolic
    - The most accessible rock was not always sought!
    - Ideal rock for architectural elements was prized

Egyptian Architecture and Engineering

- From an Old Kingdom tomb autobiography (ca 2300 BCE):

"I requested from my lord that there be brought for me a sarcophagus of limestone from Tura, and his majesty had a god's treasurer cross over together with a detachment of sailors under his charge to fetch for me this sarcophagus from Tura. It returned with him in a great cargo boat…together with its lid, a false door, a lintel, two door jambs, and one offering table."

Egyptian Architecture and Engineering

- Quarrying
  - Earliest examples: chert quarries, 40,000 years BP
  - With the centralized state, large expeditions and official quarries were organized

Egyptian Architecture and Engineering

- Quarrying
  - Limestone and sandstone quarries
    - Rock layers were identified for suitable, uniform color and texture
    - Properly spaced fractures were sought after
    - Both open air removal of hilltops and underground mines were used

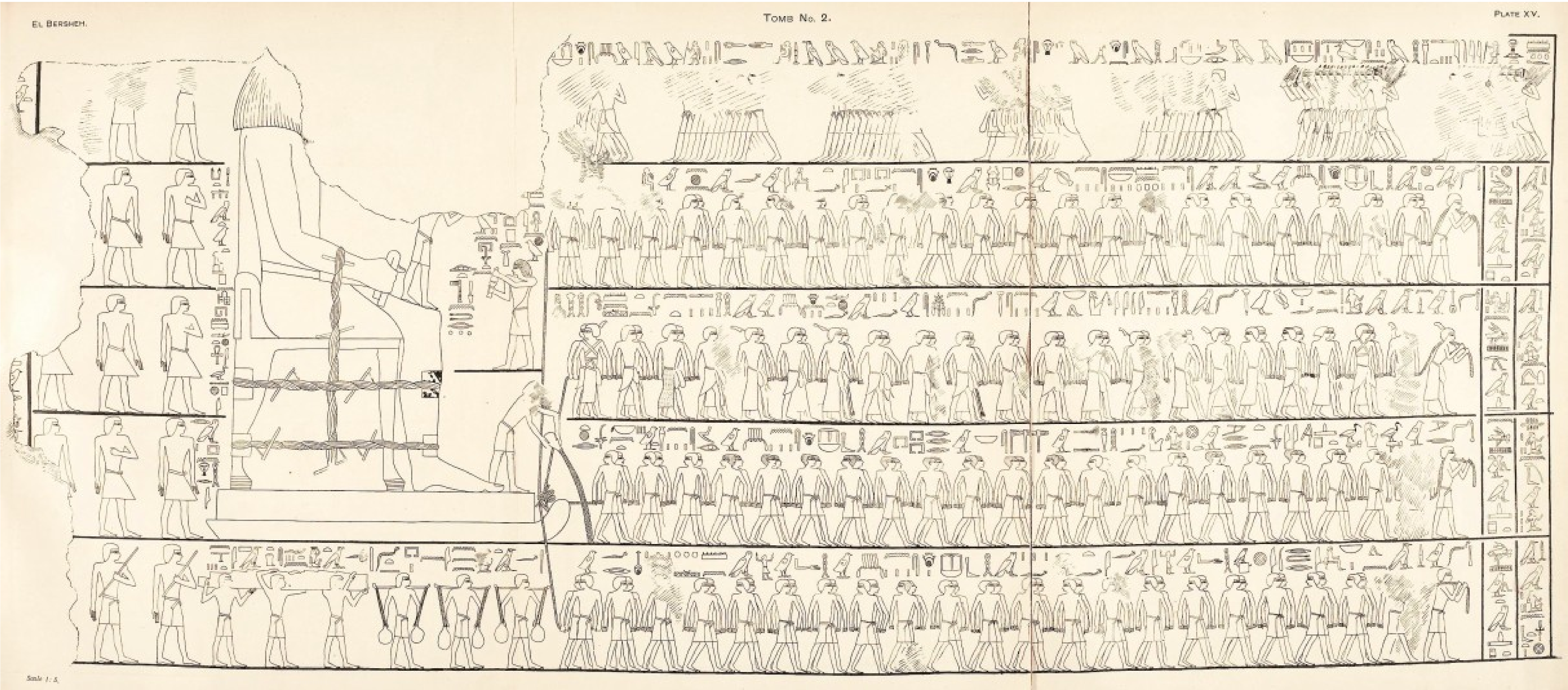
Egyptian Architecture and Engineering

- How stone was extracted:

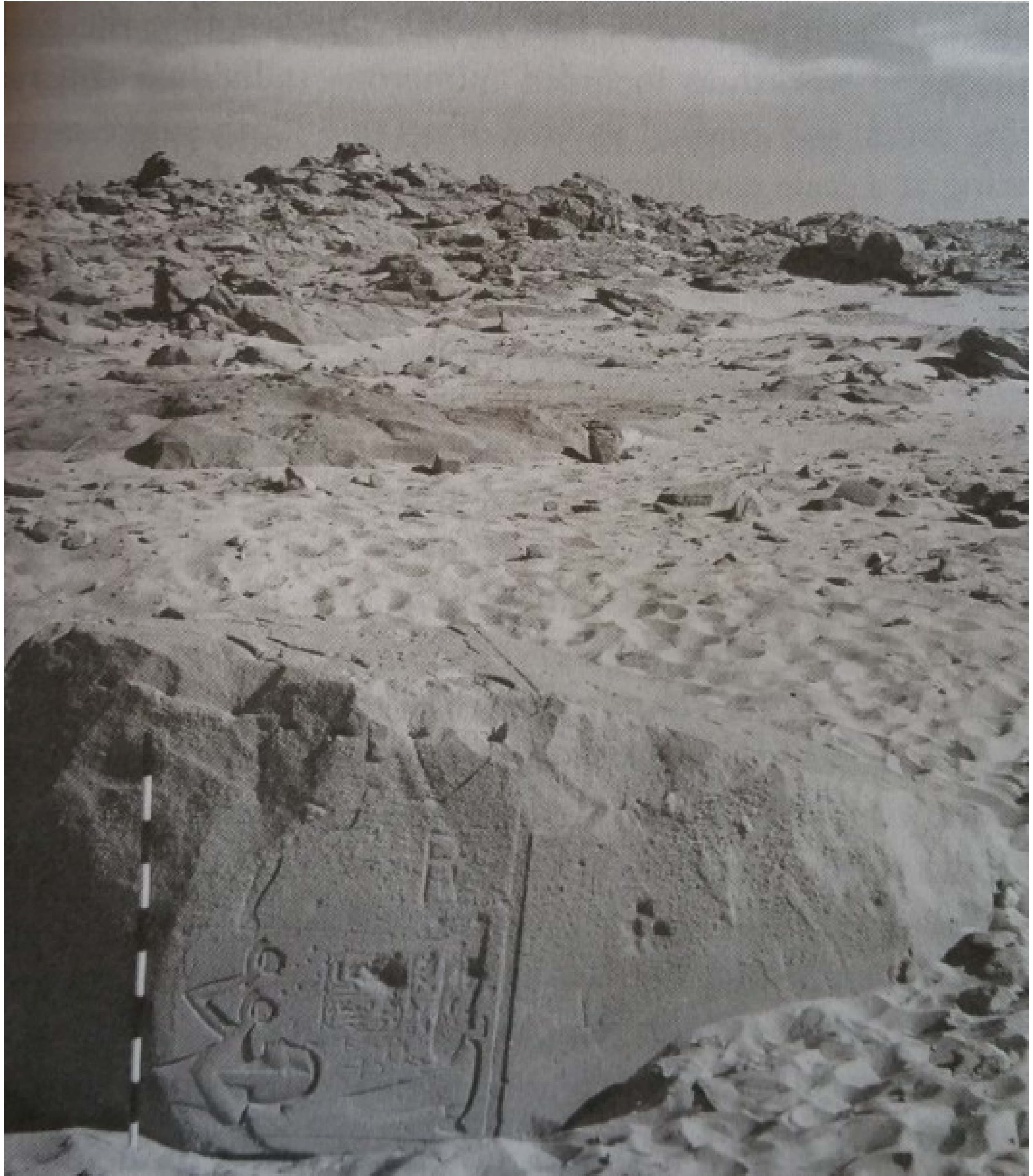
1. Rubble was removed from the vein
2. Lines were painted or chisled into the rock as guides
3. Channels were dug out with pickaxes to mark out large blocks, and to allow wooden wedges to be inserted
4. Copper saws, using water, gypsum and quartz sand, cut the rocks.
5. Wooden wedges were used to extract rock (iron was used later)

Egyptian Architecture and Engineering

- How stone was processed:
  - Often times on site.
  - Unfinished obelisks have been found at sites
  - Tomb depictions show completed statues being hauled from quarries



From the tomb of Thuthotep at Deir el-Bersha (Middle Kingdom)

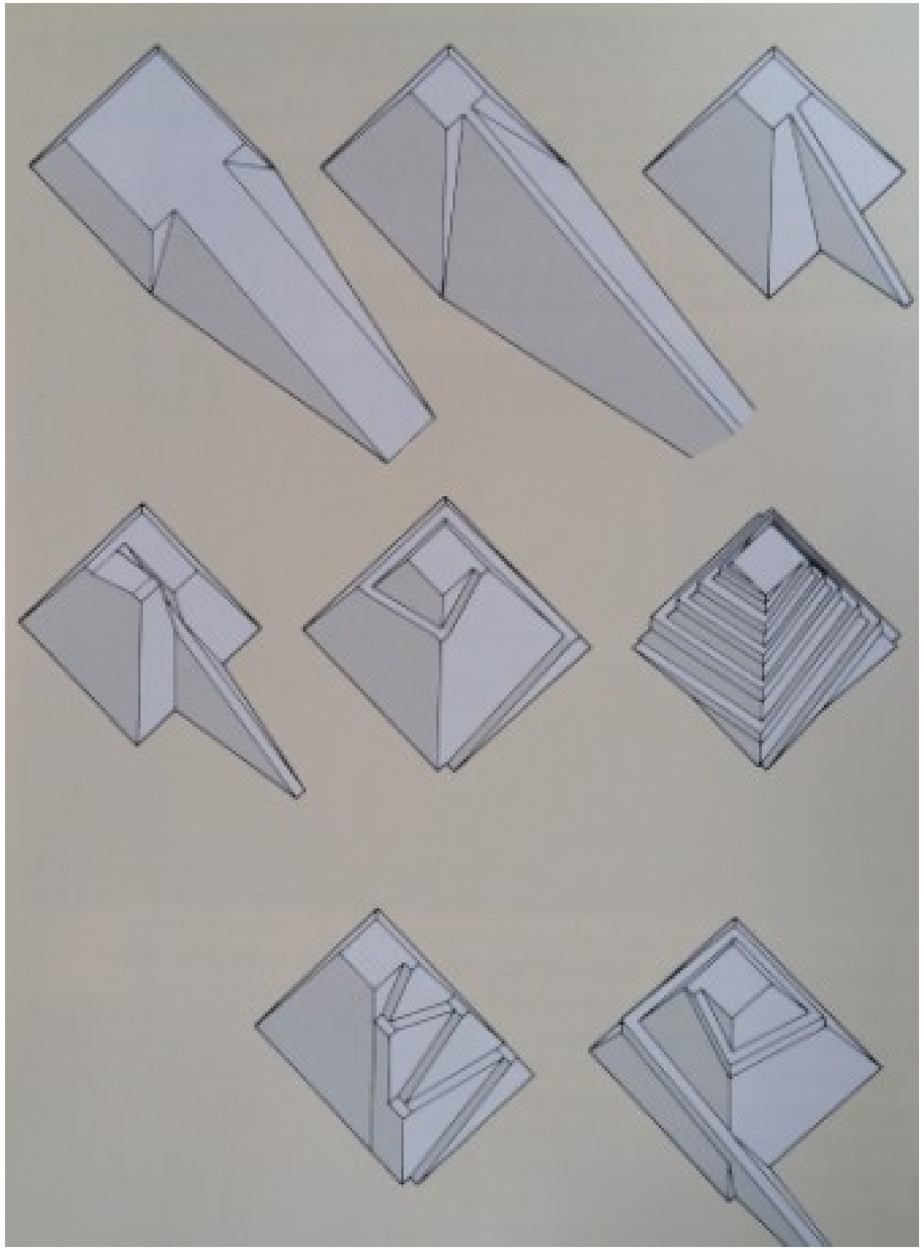


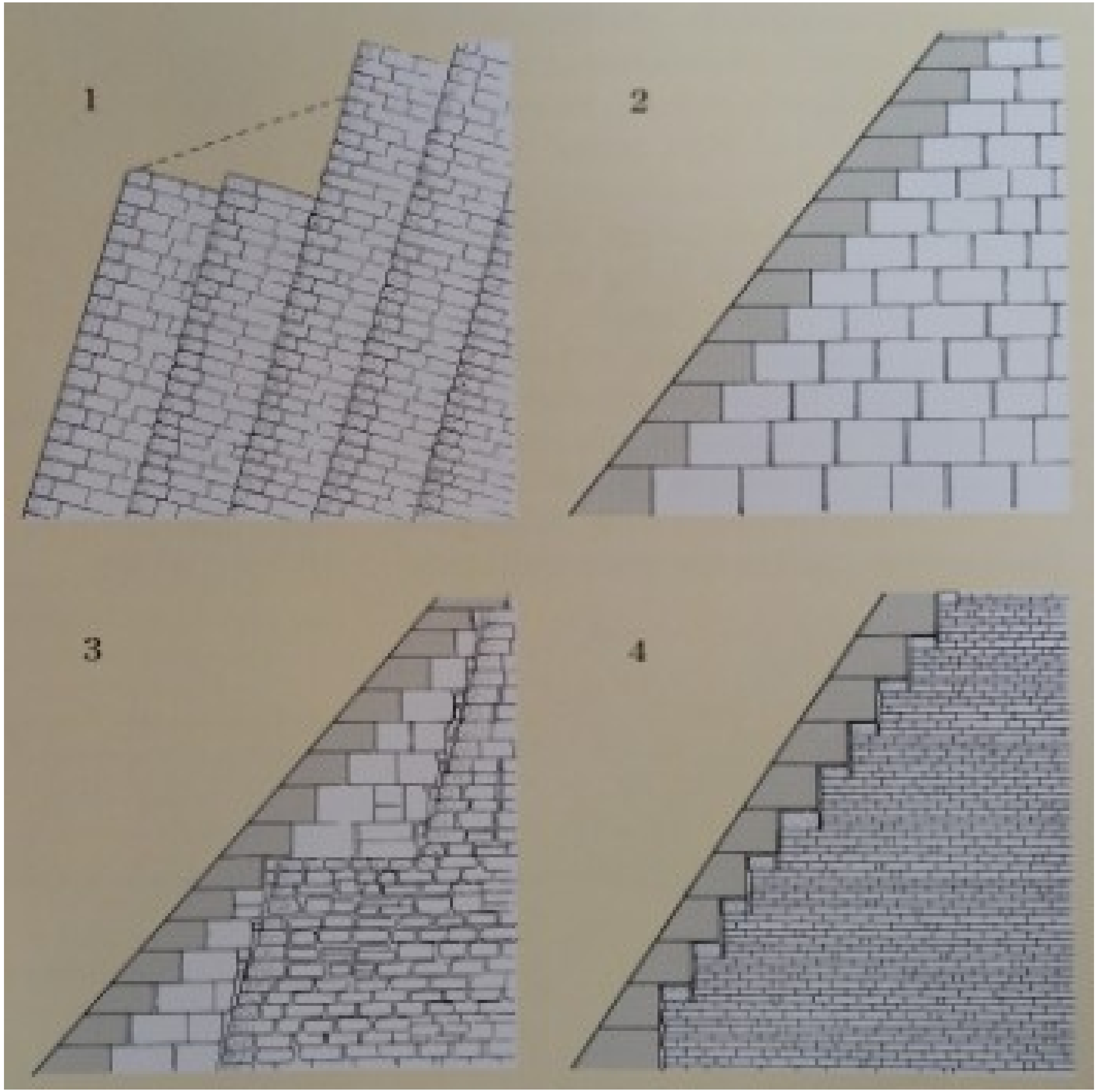
Egyptian Architecture and Engineering

- How was stone moved?
    - Smallish (< 2.5 tons): could be tumbled and flipped by 4-5 men. Heavier stones needed ropes to help.
    - Large stone: wooden sledges.
    - Sledges were pulled across large ramps made of gypsum and packed clay.
    - Rollers made of wood beams were used.
    - Artificial tracks were built to make the process more efficiently (and have bene discovered).

(See Lehner, *The Complete Pyramids*, p.208-209)


KEY
1 Great Pyramid of Khufu
2 ramps
3 housing for workers
4 supply tracks
5 harbour, canals
6 main quarry
7 Khufu's palace complex
8 causeway to valley temple
9 town
10 tombs of royal relatives and officials
11 future sites of pyramids of Menkaure and Khafre and of the Sphinx




1
2
3
4

Mathematics:
Arithmetic

Ancient Egyptian Mathematics

- The evidence we have:
  - Indirect: monuments
  - Direct: mathematical texts
  - Esp. the Rhind Mathematical Papyrus
- Unlike Mesopotamia, very few Egyptian mathematical texts have survived

Ancient Egyptian Mathematics

- The nature of the written evidence:
  - No speculation
  - Few proofs
  - Mostly school exercises and reference tables

Ancient Egyptian Mathematics

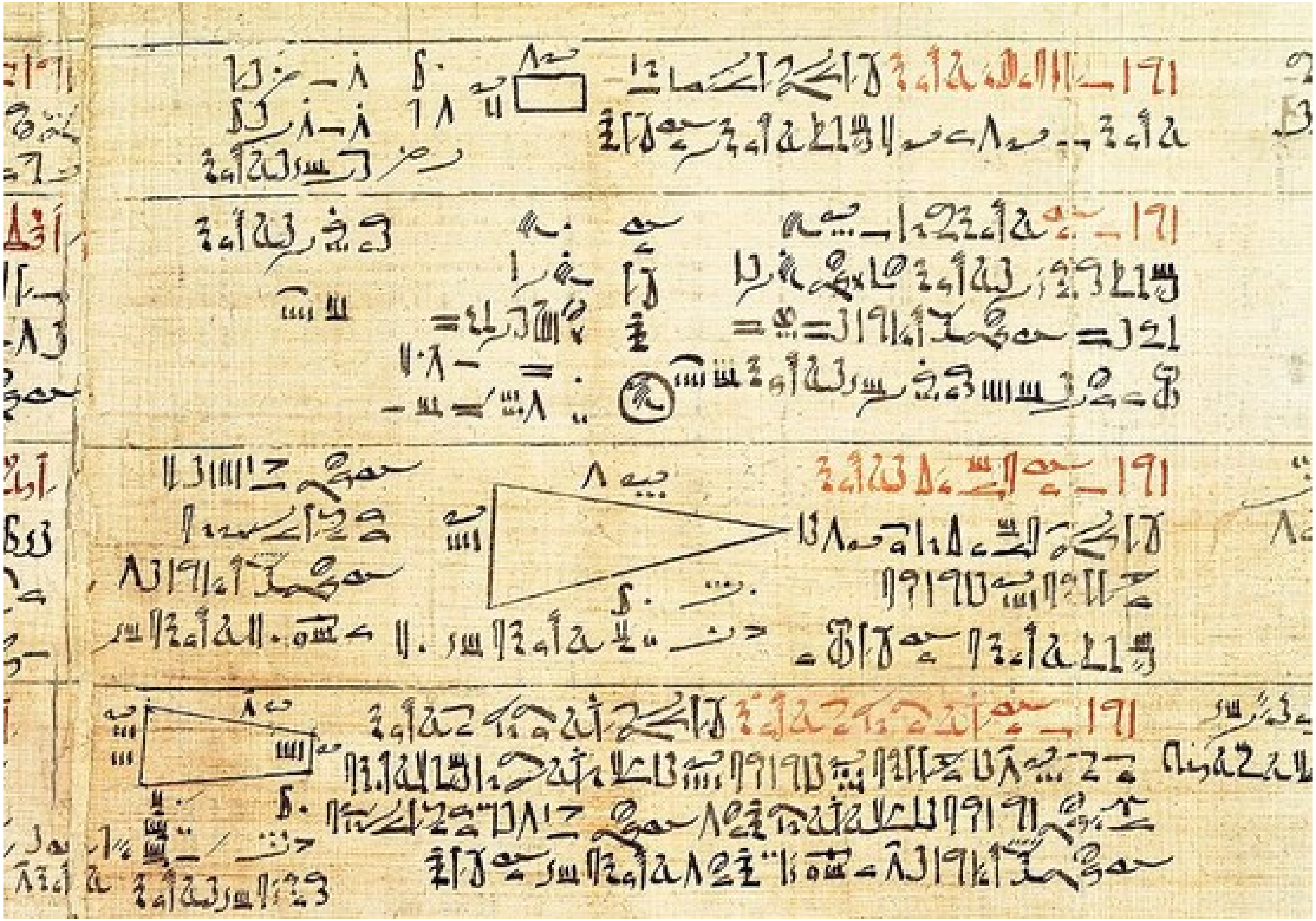
- Arithmetic
  - A skill that was crucial for scribes.
  - Used for military and engineering purposes especially:
    - Calculating rations
    - Building ramps
    - Planning a building site
    - Erecting large statues

Ancient Egyptian Mathematics

- Arithmetic
  - The basic problem: how to do calculations when you don’t have enough fingers
  - How to do calculations with combinations that are not memorized

Ancient Egyptian Mathematics

- The Rhind Mathematical Papyrus



Ancient Egyptian Mathematics

- Example problem:
  - Write the fractions 1/10 through 9/10.

$$\frac{1}{10} = \frac{1}{10} \quad ; \quad \frac{2}{10} = \frac{1}{5} \quad ; \quad \frac{3}{10} = \frac{1}{5} + \frac{1}{10}$$

$$\frac{4}{10} = \frac{1}{3} + \frac{1}{15} \quad ; \quad \frac{5}{10} = \frac{1}{2} \quad ; \quad \frac{6}{10} = \frac{1}{2} + \frac{1}{10}$$

$$\frac{7}{10} = \frac{2}{3} + \frac{1}{30} \quad ; \quad \frac{8}{10} = \frac{2}{3} + \frac{1}{10} + \frac{1}{30} \quad ; \quad \frac{9}{10} = \frac{2}{3} + \frac{1}{5} + \frac{1}{30}$$

Ancient Egyptian Mathematics

- Example problem:
  - 10 men are to share 1 loaf of bread / 2 loafs of bread / … / 10 loafs of bread. Represent with fractions the amount of bread each man receives.

Ancient Egyptian Mathematics

- Example problem:
  - Calculate the volume of a grain silo with x diameter and y height.
  - Find the area of a trapezoidal plot of land.
  - In 5 plots of land, find the area left in each when a certain section of equal length is sectioned off from each.
  - If a pyramid is x height and its base is of area A, what is its *seked*?

Ancient Egyptian Mathematics

- Arithmetic in Rhind:

47 x 33:

/1	47	/1	47
2	94	/2	94
4	188	/10	470
8	376	/20	940
16	752		
/32	1504		
Total: 33	1551	Total: 33	1551

Ancient Egyptian Mathematics

- Arithmetic in Rhind:

1/33 * 47 … or 47 ÷ 33

The Egyptians said: “Treat 33 so as to obtain 47”

So, algebraically: 33x = 47

Ancient Egyptian Mathematics

- Arithmetic in Rhind:

47 ÷ 33

/1	33
/$\overline{3}$	11
/$\overline{11}$	3
Total: 1+$\overline{3}$+$\overline{11}$	47

Ancient Egyptian Mathematics

- Geometry
  - Most accurate value for π until the Greeks: 3.16 (.06% error)
  - Known because Rhind reproduces a formula for determining the area of a circle:
    - Subtract 1/9th of the diameter, then square the result.
    - With d = 10:
      - $A = (d - d/9)^2 = 79.0123\ldots$

$A = \pi r^2 = 78.5398\ldots$

-
    - The Egyptian calculation is 99.4% accurate!

Ancient Egyptian Mathematics

- How did the Egyptians determine pi so accurately?
  - Archimedes used circumscribed and inscribed polygons
- The Egyptians did not know pi *per se* nor calculate it
- They simply used the number 9, and it had good results.

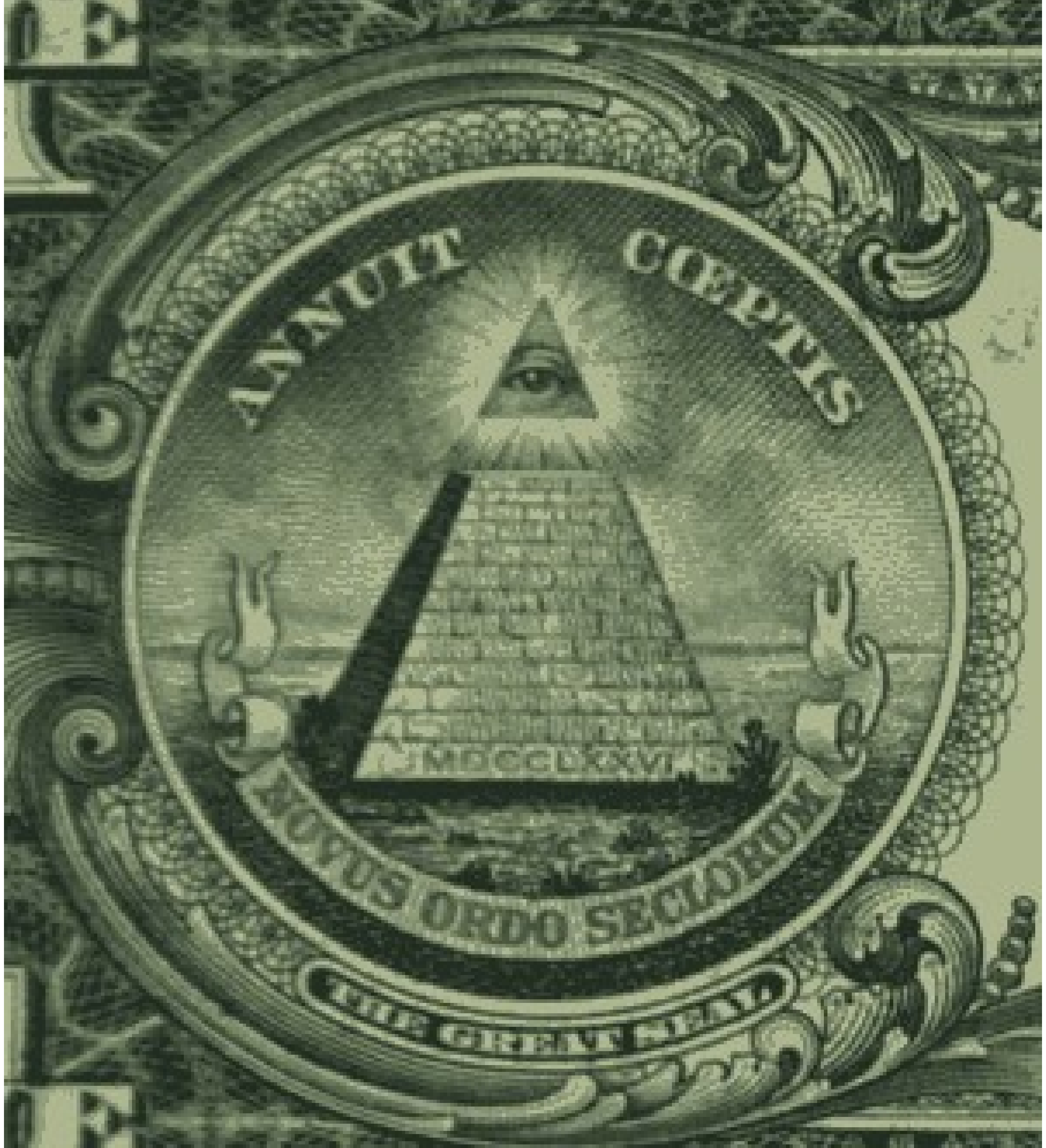
- Moral: estimation can go a long ways!

Ancient Egyptian Mathematics

- Pythagorean triples:
  - The Old Kingdom pyramids suggest they understood what a 3:4:5 right triangle was.
- Why would this knowledge be useful?

Ancient Egyptian Mathematics

- The volume of a frustrum:


ANNUIT COEPTIS
MDCCLXXVI
NOVUS ORDO SECLORUM
THE GREAT SEAL

Ancient Egyptian Mathematics

- The volume of a frustrum:
  - They used formula equivalent to:

$$V = \frac{1}{3}h(a^2 + ab + b^2).$$

- Unknown how they arrived at this. They must have understood:
  - The height of the two pyramids implied by a frustrum are proportional to their bases
  - How to solve a geometric problem by changing or inscribing the problematic figure